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ON THE FOCAL LENGTH OF MICROSCOPIC OBJECTIVES.

CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.*

BY CHAS. R. CROSS.

THE investigation of which the present article is a summary, was undertaken in order to see if some reliable method of measuring the focal length of microscope objectives could not be found. The importance of such a method will be apparent to all who have had occasion to make use of objectives by different makers. The focal length of lenses of the same denomination is subject to so great a variation that comparison of these by means of their assumed focal lengths too often gives no true idea of their relative excellence. For example, if two quarter-inch objectives be compared, and one gives results much superior to that given by the other, we cannot be at all sure that the better lens is not really of shorter focus than its designation would indicate.

The question immediately arises, what is the focal length of a compound objective? The focal length of a simple lens, or of a system of lenses in actual contact, is the distance from the optical centre of that lens or system to its principal focus. But as a system of lenses not in contact, like the triplet objective, has no optical centre, the term is only a general appellation serving to group together objectives of approximately the same magnifying power. If every system of lenses possessed an optical centre, or could be replaced by a single lens, we might define the focal length of such

* In a recent number of this *Journal* (March, 1870, p. 208,) attention is called to the great need that is felt of a laboratory for physical research. In 1864, President Rogers, in his plans of this Institute, proposed such a laboratory; and, during the past year, we have fitted up rooms in which our regular students verify many of the laws and measure the principal constants of physics, while more advanced pupils carry on original investigations. In developing the first part of this plan, many unexpected results have been arrived at, and, thinking that they may be of value elsewhere, I hope to make them the subject of a future paper. I think the accompanying article by Mr. Cross will show, that although we cannot always expect to have students with his skill and perseverance, yet that such a laboratory ought to furnish many additions to physical science. E. C. PICKERING.

a system as being the focal length of a single lens equivalent to the system in magnifying power. But, as this is not the case, a lens replacing the system when the conjugate foci are separated from each other by a distance, d, will not replace it if the foci are separated by any other distance, d^1; and the difference in focal length of lenses replacing the system under these different circumstances varies with the internal arrangement of the system. But for any constant distance between the conjugate foci any system can be replaced by an equivalent single lens; and in order to attach a definite meaning to the term "focal length," as applied to triplet objectives, I would propose that the focal length of such an objective be understood as being the focal length of a lens replacing the system when the distance between the conjugate foci is ten inches.

The present series of experiments was begun upon the supposition that a triplet microscope objective has an optical centre, and the formulæ applied to obtain the focal length were based on this supposition, which, on further investigation, proved to be incorrect. Observations recorded in the sequel showed, however, that though not exactly correct, the formulæ offer a very close approximation to the truth for distances of about ten inches between the conjugate foci; the variation in the focal length of the lens replacing the system corresponding to a difference of several inches in that distance being very small.

We may, then, deduce a formula by which to find an equivalent simple lens from the relations between the principal focus, the conjugate foci, and the relative magnitude of object and image for given distances between them.

The principal focus of a simple lens can be determined from the formulæ, $\frac{1}{p} + \frac{1}{p^1} = \frac{1}{f}$, in which p and p^1 are the conjugate force obtained by direct experiment, and f is the principal focus sought. Even if the triple objective had an optical centre, this formulæ could not be applied directly, owing to the practical impossibility of finding this centre, which, moreover, would change with the relative change of position of the lenses composing it in varying the adjustment for different thicknesses of the glass covering the object. The direct use of the formulæ would require the distances p and p^1 to be known. Some modification must, therefore, be adopted.

The method ordinarily in use by the makers seems to be to grind

the lenses with certain radii, which are assumed to give approximately definite "focal lengths." The glasses, if tested at all, are compared with some standard objective, by means of their magnifying powers with the same eye-piece, a method liable to considerable errors.

The method which I use is based upon two equations, the one given above, $\frac{1}{p}+\frac{1}{p^1}=\frac{1}{f}$ (1) and $\frac{p}{p^1}=\frac{\text{size of object}}{\text{size of image}}=n$ (2), in which n is the ratio of the size of the object to the size of the image, the conjugate foci being p and p^1. It is clear that though we cannot measure p and p^1 separately, we can measure their sum, that is the distance between the object and the image, which we call l; n can be found by measuring the size of the image of an object of known magnitude, a finely divided scale throwing this image on a second divided scale.

We have from (1) $p^1f+pf^1=pp^1$, or $lf=pp^1=(l-p^1)p^1$ (3), as $p+p^1=l$. But $p=np^1$ from (2); therefore, $p^1+np^1=l$, and $p^1=\frac{l}{n+1}$ (4). Combining (3) and (4) $fl=\frac{nl^2}{(n+1)^2}$, and $f=\frac{nl}{(n+1)^2}$. If, therefore, we find n and l, we can easily find f, the focal length of the equivalent lens.

Two sets of experiments were made with a somewhat different arrangement of apparatus in each. In both the image of a glass scale divided to $\frac{1}{100}$ millimetres ($\frac{1}{2500}$ inch) was thrown upon an engine-divided paper scale ($\frac{1}{50}$ or $\frac{1}{60}$ inch), the image and the paper being in the focus of a double-convex lens used as an eye-piece, so that the size of the magnified image was read directly on the paper scale, estimating by the eye to tenths of the divisions. The distance from the glass to the paper was measured with a steel rule graduated to millimetres. The magnitude of the image varies so slowly for any variation of l that this was taken only in whole millimetres. Any error in the measurement would be less perceptible in the result, the shorter the focus of the lens measured.

The first form of apparatus consists of a stand with a vertical block pierced with two holes, in one of which is placed the objective to be measured with its optical axis horizontal. Through the other hole (the lower one) slides a horizontal bar, at one end of which is the micrometer used as the object, at the other end the paper scale on which the size of the image is measured, reading with an eye-piece detached from the instrument. This bar is

moved in either direction until the image is thrown directly on the paper scale, the motion being accomplished by a screw giving a fine adjustment, while the nut in which the screw plays is moved by the hand for the coarse adjustment. The length, l, between the two scales was measured once for all.

The results given by this arrangement were very satisfactory, but owing to its horizontal position and a difficulty in centering the lenses, I afterwards adopted a somewhat different apparatus, with which most of the recorded observations were made. Here I employ a microscope stand, using simply the tube and stage, placing the objective in its usual position, and throwing the image of the micrometer upon the paper scale. The latter is now gummed to the end of the tube usually occupied by the eye-piece, which is replaced by a convex lens detached from the instrument as before. The distance, l, now a variable, has to be measured with each objective. This arrangement can evidently be applied to all microscopes, rendering it possible for any one to determine the focal length of his own objectives with the greatest ease.

The following measurements will show the accuracy of the method:—

Tolles, Second Quality ¼.				Smith and Beck, ¼.			
Apparatus No. 1.		Apparatus No. 2.		Apparatus No. 1.		Apparatus No. 2.	
l = 311 mm.		l = 246 mm.		l = 311 mm.		l = 253 mm.	
f.	N.	f.	N.	f.	N.	f.	N.
·2416	20	·2424	30	·2115	20	·2103	40
·2416	10	·2419	50	·2112	30	·2097	40
·2416	20	·2415	50	·2107	30	·2097	40
·2397	20	·2415	40	·2107	30		
M = ·2411		M. = ·2418		·2107	30	M. = ·2099	
P. E. = ·00029		P. E. = ·00011		·2107	30	P. E. = ·00011	
p. e. = ·00058		p. e. = ·00022		·2104	25	p. e. = ·00019	
				·2086	20		
				·2086	20		
				·2079	20		
				M. = ·2095			
				P. E. = ·00025			
				p. e. = ·00084			

The preceding measurements were made with both forms of apparatus, and at an early period of the investigation; the following

are by the second method alone, and were taken after considerable practice in using the apparatus.

Hartnack, No. 9.		Tolles, $\frac{1}{12}$.		Nachet, No. 7.	
Adjusted for		Adjusted for		Adjusted for	
Object Covered.	Object Uncovered.	Object Covered.	Object Uncovered.	Object Covered.	Object Uncovered.
l = 248 mm.	l = 249 mm.	l = 257 mm.	l = 258 mm.	l = 265 mm.	l = 265 mm.
f. N.	f. N.	f. N.	f. N.	f. N.	f. N.
·0819 10	·0917 10	·0640 10	·0900 15	·0721 10	·0898 15
·0816 10	·0911 10	·0636 10	·0898 15	·0721 10	·0898 15
·0816 10	·0910 10	·0636 10	·0898 15	·0718 10	·0897 10
M. = ·0817	·0910 15	M. = ·0638	M. = ·0899	·0718 10	·0896 15
P. E. = ·00006	M. = ·0912	P. E. = ·00001	P. E. = ·00004	M. = ·0719	M. = ·0897
p. e. = ·00010	P. E. = ·00009	p. e. = ·00002	p. e. = ·00007	P. E. = ·00005	P. E. = ·00003
	p. e. = ·00019			p. e. = ·00010	p. e. = ·00006
l. = 249 mm.	l = 264 mm.				
f. N.	f. N.				
·0817 10	·0923 10				
·0817 10	·0914 10				
·0817 10	·0911 10				
·0817 10	M. = ·0915				
·0816 10	P. E. = ·00020				
M. = ·0817	p. e. = ·00035				
P. E. = ·00002					
p. e. = ·00006					

In the two preceding tables, the focus (f) is given in decimals of an inch, as computed from each reading. N is the number of

spaces observed on the micrometer scale, and l the distance between the two scales. The most probable mean (M) is computed by giving weight to each observation proportional to the number of spaces (N) taken; that is, assuming that the whole error lies in reading the size of the magnified image on the paper scale. P. E. is the probable error of the mean (M) and *p. e.* that of one observation.

In all the measurements, the objective was refocussed for each reading, and in those given above it was dismounted and remounted for each set.

The table below gives the results of several hundred measurements on various objectives. The first column gives the order in the table for convenience of reference; the second the name of the maker and designation as given by him; the column headed A gives the adjustment, whether for covered or uncovered glass, with the position of the index, if this is present; also whether the lens is wet or dry if an immersion lens. The next column gives in millimetres the distance denoted in the formula by l; that is, the distance between the two scales. The value of f is in all cases the most probable mean of a number of observations. The column headed B gives the focal length indicated by the maker in decimals of an inch. The next column, headed "diff. A.," gives the difference between the actual focal length, as determined by these observations, and the designated focal length as given in column B. The last column, headed "diff. B.," gives the difference in decimals of an inch between the *extreme* values of f as given from different observations by this method.

The micrometer used as an object, was a glass ruled scale divided to $\frac{1}{100}$ mm. ($\frac{1}{2500}$ inch,) except in the objectives numbered 9, 10, 16, 18, 28, 31, 32 (marked with an asterisk) in which a $\frac{1}{1000}$ in. stage micrometer was substituted, as the more finely divided scale could not be read with such low powers as these objectives gave.

Objectives Nos. 16 and 28 are single lenses; the rest are triplets. Lens No. 32, by Zentmayer, had been slightly altered in focal length to adapt it to a gas microscope. The adjustment for covered object shortens the focus of all the glasses examined, except Nos. 11, 12 and 17, in which a lengthening of focus takes place. No. 11 is adjusted for glass covering when the index is at O, contrary to the usual method in which the greater the number of the index the thicker the covering for which the objective is adjusted. Objective No 12 is No. 11 with the front lens removed and an "immersion front," a lens of different focus screwed on in its place.

No.	Name of Maker.	A	l	Focal Length.		B	diff. A.	diff. B.
			mm.	mm.	in.	in.	in.	in.
1 a	Hartnack, No 10.	Unc. Dry.	249	1 985	·0782			·0006
b		Cov'd Dry.	249	1·696	·0668			·0009
c		Unc. Wet.	249	1·991	·0784			·0002
d		Cov'd Wet.	249	1·681	·0662			·0002
2	Hartnack, No. 9.	Unc. Dry.	249	2·317	·0912			·0007
		Unc. Dry.	264	2·325	·0915			·0012
		Cov'd Dry.	248	2·079	·0818			·0003
		Cov'd Dry.	249	2·073	·0816			·0001
		Cov'd Dry.	261	2·079	·0817			·0006
		Unc. Wet.	261	2·339	·0921			·0007
3	Hartnack, $\frac{1}{4}$ in.		249	6·295	·2478	·2500	— ·0022	·0013
4	Hartnack, $\frac{1}{2}$ in.		259	13·659	·5337	·5000	+ ·0337	·0027
5 a	Nachet, No. 7.	Unc'd.	265	2·279	·0897			·0002
b		Cov'd.	265	1·827	·0719			·0003
6	Nachet, No. 5.		264	3·066	·1207			·0004
7	Nachet, No. 3.		235	4·666	·1758			·0004
8	Nachet, No. 2.		239	6·469	·2547			·0006
9*	Nachet, No. 1.		244	14·791	·5823			·0012
10*	Nachet, No. 0.		268	39·103	1·5395			·0027
11 a	Ross' $\frac{1}{12}$ in.	0	249	1·675	·0659			·0004
b		22	250	1·900	·0748	·0833	— ·0085	·0013
12 a	Ross' $\frac{1}{12}$ in., with Immersion	0 Dry.	249	1·265	·0498			·0001
b	Front by Tolles.	0 Wet.	249	1·273	·0501			·0006
c		31 Dry.	250	1·696	·0668			·0005
d		31 Wet.	250	1·689	·0665			·0006
13	Smith and Beck, $\frac{1}{5}$ in.		255	4·906	·1931	·2000	— ·0069	·0017
14 a	Smith and Beck, $\frac{1}{4}$ in.	0	253	5·332	·2099	·2500	— ·0401	·0006
b		18	253	5·038	·1983			·0005
15	Smith and Beck, $\frac{2}{3}$ in.		257	16·330	·6429	·6667	— ·0238	·0000
16*	Smith and Beck, $1\frac{1}{2}$ in.	Single Lens.	276	33·990	1·3382	1·5000	— ·1617	·0033

No.	Name of Maker.	A	l	Focal Length.		B	diff. A.	diff. B.
			mm.	mm.	in.	in.	in.	in.
17 a	Collins, $\frac{4}{10}$ in.	3	258	8·180	·3220			·0009
b		20	259	8·502	·3347	·4000	— ·0653	·0010
18*	Collins, $1\frac{1}{2}$ in.		271	42·572	1·6761	1·5000	+ ·1761	.0047
19 a	Spenser, $\frac{1}{2}$ in.	Unc'd.	266	9·717	·3826	·5000	— ·1174	·0000
b		Cov'd.	266	8 947	·3522			·0018
20 a	Tolles, "$\frac{1}{12}$ in. dry, $\frac{1}{15}$ in. wet."	0 Dry.	258	2 286	·0899	·0833	+ ·0066	·0002
b		19 Dry.	257	1·620	·0638			·0004
c		6·3. Dry.	256	2·024	·0797			·0012
d		6·3. Wet.	256	1·995	·0786	·0667	+ ·0119	·0003
21	Tolles, $\frac{1}{12}$ in.	0 Dry.	257	2·213	·0871	·0833	+ ·0038	·0011
22	Tolles, $\frac{1}{10}$ in.	0 Dry.	257	2·898	·1141	·1000	+ ·0141	·0014
23 a	Tolles, $\frac{1}{10}$ in.	0 Dry.	258	2·874	·1131	·1000	+ ·0131	·0000
b		14 Dry.	259	2·283	·0899			·0008
c		0 Wet.	258	2·910	·1146	·1000	+ ·0146	·0005
d		14 Wet.	259	2·263	·0891			·0004
24 a	Tolles, $\frac{1}{8}$ in.	0	258	3·217	·1267	·1250	+ ·0017	·0011
b		$16\frac{1}{2}$	258	2·683	·1056			·0005
25 a	Tolles, $\frac{1}{4}$ in.	0	263	6·083	·2395	·2500	— ·0105	·0011
b		10	263	5·675	·2234			·0026
26	Tolles, $\frac{1}{4}$ in., 2d Q.		246	6·143	·2418	·2500	— ·0082	·0009
27	Tolles, $\frac{1}{4}$ in., 2d Q.		248	6·400	·2520	·2500	+ ·0020	·0010
28*	Tolles, 1 in., 2d Q.	Single Lens.	259	25·188	·9916	1·0000	— ·0084	·0000
29 a	Wales, $\frac{4}{10}$ in.	0	258	6·818	·2684	·4000	— ·1316	·0008
b		9	258	6·563	·2584			·0007
30	Zentmayer, $\frac{4}{10}$ in.		255	9·193	·3619	·4000	— ·0381	·0009
31*	Zentmayer, $\frac{8}{10}$ in.		269	20·113	·7918	·8000	— ·0082	·0000
32*	Zentmayer, $1\frac{1}{2}$ in.		280	36 839	1·4504	1·5000	— ·0496	·0022
33 a	Grunow, $\frac{1}{4}$ in.	Unc'd.	273	5·253	·2068	·2500	— ·0432	·0012
b		Cov'd.	273	5 059	·1992			·0003
34	Grunow, $\frac{1}{2}$ in.		255	8·902	·3505	·5000	— ·1495	·0021

The contents of column A may need some further explanation. The immersion lenses were measured both wet and dry to determine the change of focal distance, which is very small unless the interior arrangement of the lenses is altered by moving the index circle. The figures in this column are the numbers given by the index if there was one; if no index was attached to the objective the extremes of the adjustment were taken, and are indicated by the words "Unc." for uncovered, and "Cov'd" for covered adjustment. The $\frac{1}{100}$ millimetre scale used as the object was always uncovered, whether the lens was adjusted for covered and uncovered objects, as no other method seemed so generally applicable. This, of course, rendered the definition somewhat indistinct when the lens was adjusted for glass covering, which will explain the greater difference in the corresponding extreme values of f in the last column.

The measurements in the preceding table were made with the second form of apparatus, so that the length of l varies slightly from the normal length of 10 inches. The extreme values are 276 mm. (10·87 inches) for No. 16 and 235 mm. (9·25 inches) for No. 7. To ascertain the effect of this variation on f, the computed focal distance, the following observations were made. 1st. A Smith and Beck $\frac{1}{4}$ inch (No. 14) was measured, first with $l = 279$ mm. (10·98 inches), and then with $l = 412$ mm. (16·22 inches). The computed values of f in the two cases were—

$$l = 279 \text{ mm.}, f = \cdot 2102 \text{ in.}; \; l = 412 \text{ mm.}, f = \cdot 2035 \text{ in.},$$

giving a difference of only ·0067 inch in f for a difference of 133 mm. (5·24 inches) in l. 2d. A Tolles second quality $\frac{1}{4}$ inch (No. 26) was measured in the same way, giving values of f as follows:

$$l = 259 \text{ mm.}, f = \cdot 2424 \text{ in.}; \; l = 414 \text{ mm.}, f = \cdot 2395 \text{ in.}$$

a difference of but ·0029 inches, corresponding to a difference of 155 m m. (6·10 inches) in l. From these results it was inferred that with the maximum deviation (in No. 16) of 22 m m., (0·87 in.) from the normal value of l, the correction required to reduce the value of f to that standard length, would be within the limits of probable error, and in most of the objectives the deviation of l is far less than in this case.

An examination of the table will show that the focal length of the objectives of some makers differs considerably from the length marked upon them. For example, No. 34 marked $\frac{1}{2}$ inch is really

a $\frac{1}{3}$ inch objective; No. 33 marked $\frac{1}{4}$ inch is really a $\frac{1}{5}$ inch; No. 29 marked $\frac{4}{10}$ inch is a $\frac{1}{4}$. Lens No. 14 marked $\frac{1}{4}$ inch is really a $\frac{1}{5}$ inch; but Nos. 13, 15, by the same makers, are correctly designated $\frac{1}{5}$ inch, $\frac{2}{3}$ inch. Differences of this kind must of necessity lead to a great confusion in comparing objectives with one another. I would therefore suggest that each objective made should be measured before being offered for sale, that this confusion may cease to exist. A convenient arrangement would be to fix a glass scale divided to $\frac{1}{50}$ or $\frac{1}{100}$ inch in the draw-tube, sliding in the tube of the microscope, and measure as I have already described. The draw-tube should be moved till the front of the ruled glass shall be exactly 10 inches from the micrometer used as the object. Or it would be more convenient still to have an apparatus similar to the first form, but arranged with a suitable stage and stand so that it can be set at any desired angle. The distance 10 inches (254 mm.), suggested as a standard is chosen because it is the normal distance of distinct vision, as well as about the length used by microscopists in actual work.

An inspection of the formula $f = \frac{n\,l}{(n+1)^2}$ shows (1) that the focal length of any lens is not inversely proportional to its magnifying power with a given distance (l) between the conjugate foci, as is commonly assumed, but to $\frac{n}{(n+1)^2}$, the ordinary supposition approaching absolute correctness as n increases. Hence the inaccuracy of any system of estimating focal lengths upon this assumption when applied to lenses of long focus.

(2.) The shorter the focal length of the objective, the less will any error in the measurement of l affect the result.

(3.) Any error in the measurement of n also affects the result less in a lens of short focus. It would therefore appear that by this method the most accurate results are obtained with the objectives of highest power. The following examples from the records of my observations will illustrate this last point. The numerators of the fractions are the readings on the paper scale, the denominators, the number of spaces of the micrometer scale corresponding to these readings, the quotient being of course the value of one of these magnified spaces in fiftieths of an inch.

With objective No. 16*, Smith and Beck, $1\frac{1}{2}$ inch—

Scale readings $\frac{32\cdot8}{11} = 2\cdot982$. $\frac{32\cdot7}{11} = 2\cdot973$. $\frac{32\cdot7}{11} = 2\cdot973$.

Give values of $n = 5\cdot964$. $5\cdot946$. $5\cdot946$.

The corresponding values of f are—

1·3362 inch. 1·3392 inch. 1·3392 inch.

$l = 276$ mm. M = 1·3382 in. Differences of extremes = ·0030 in.

With objective No. 8, Nachet No. 2.

Scale readings $\frac{41\cdot2}{60} = \cdot6866$. $\frac{41\cdot2}{60} = \cdot6866$. $\frac{34\cdot4}{60} = \cdot6880$. $\frac{41\cdot3}{60} = \cdot6883$

Give values of $n = 34\cdot879$. $34\cdot879$. $34\cdot950$. $34\cdot965$.

The corresponding values of f are—

·25494 inch. ·25494 inch. ·25446 inch. ·25435 inch.

$l = 239$ mm. M = ·25466 in. Difference of extremes = ·00059 in.

With objective No. 1, *c*, Hartnack, No. 10.

Scale readings $\frac{36\cdot4}{15} = 2\cdot447$. $\frac{24\cdot2}{10} = 2\cdot420$. $\frac{36\cdot3}{15} = 2\cdot420$. $\frac{36\cdot3}{15} = 2\cdot420$.

Give values of $n = 123\cdot29$. $122\cdot94$. $122\cdot94$. $122\cdot94$.

The corresponding values of f are—

·07824 inch. ·07845 inch. ·07845 inch. ·07845 inch.

$l = 249$ mm. M = ·07839 in. Difference of extremes = ·00021 in.

The increasing change in f for the same variation of the scale reading is clearly seen on comparing the above sets of observations. The diminution of the number of divisions measured in a short focus objective is a partially neutralizing circumstance, which can, however, be avoided by using a lens of long focus for the eye-piece, so as to gain a larger field of view.

The chief difficulty met with in pursuing this research was that of procuring a suitable scale for the object, the image of which was to be meaured. In the earliest measurements a scale on glass ruled to $\frac{1}{1000}$ inches was used, but the lines were jagged at the edges, their breadth was variable and their spacing unequal. Next an eye-piece micrometer belonging to a Smith and Beck's microscope was used, the divisions being $\frac{1}{200}$ inch, but though this was an improvement on the former the results were still unsatisfactory. Finally a micrometer reading to $\frac{1}{100}$ millimetres was used, which was all that could be desired in clearness and evenness of lines and equality of spacing. In some cases, for long focus objectives a $\frac{1}{1000}$ inch micrometer scale was substituted, as before stated. The paper scale used to measure the size of the image was divided to $\frac{1}{50}$ or $\frac{1}{60}$ of an

inch, while these divisions were sub-divided by the eye into tenths, giving the measurement to $\frac{1}{500}$ or $\frac{1}{600}$ of an inch. The measurements can in general be relied upon within $\frac{1}{500}$ or $\frac{1}{600}$ of an inch. A scale divided into fractions of a millimetre would have been preferable had such been easily procurable, as saving labor in the calculation of the focal lengths.

Difficulty was at first apprehended from the expansion of the paper scale, especially as this was attached to the microscope tube by mucilage, but measurements made at different times both when moist and when dry showed no appreciable variation to be ascribed to that cause. A steel scale was at one time substituted, but being less easily read, its use was abandoned. A glass scale divided to half millimetres would be valuable in an extensive application of this method, because of the greater ease of estimating to tenths with narrow lines, those on printed scales being necessarily somewhat coarse.

The present investigation has suggested an inquiry into the laws of the foci of systems of lenses as related to variations in the distances between them, which, in connection with the law of variation of focal length with varying values of l, it is my intention to make the subject of further research.

I would take this opportunity of expressing my obligations to the gentleman who have so kindly allowed me the use of their objectives, and especially to Mr. Charles Stodder of the Boston Optical Works for the large number of triplet lenses, by various makers, which he has placed at my disposal.

It is hoped that the method described in this paper may be found of value as offering a ready and reliable method of measuring focal lengths, so that there may be no necessity of doubt as to the true focal length of any objective.

Boston, May 3d, 1870.